$J_1 = J_2 = h = v = 0$ in the Hamiltonian (1). In the case of the F model, for example, it is known that the inclusion of a nonzero field (h, v) changes the infinite-order transition to a second-order one.[7] The inclusion of some nonzero values for J_1 and J_2 could have the same consequence in the present problem. It does appear safe, however, to infer that the inclusion of the four-spin interactions will in general *not* result in $\alpha = \alpha' = 0$.

The result that the nearest-neighbor square Ising lattice is a singular case with $\alpha = \alpha' = 0$ also appears somewhat disturbing, for it is generally believed that the critical exponents should depend only on the dimensionality of the model, and not on the range of interactions. We wish to present some counter arguments. First, some information is available at one particular point of the parameter space, namely, $J_1 = J_2 = J = J'$ and $J_4 = 0$. This is the square Ising lattice with equivalent first- and second-neighbor (crossing) interactions. For this model Domb and Dalton[8] and Dalton and Wood[9] have carried out numerical analyses on the high- and low-temperature series expansions. The study on the high-temperature series led to the critical exponent[8]

$$\gamma \cong 1.75 \ , \tag{8}$$

which does not differ from that of the nearest-neighbor planar Ising lattices. On the other hand, the study on the low-temperature series did not lead to such agreement. The authors of Ref. 9 attributed their results on the low-temperature exponents β and γ' to the erratic behavior of the Padé approximants. On reexamining their data on the first- and second-neighbor square lattice, we feel that unless something drastic happens in the high Padé approximants, it should be safe to infer the following bounds on the critical exponents β and γ':

$$0.80 < \gamma' < 1.30, \quad 0.13 < \beta < 0.16 \ . \tag{9}$$

Accepting (9), the Rushbrook inequality $\alpha' + 2\beta + \gamma' \geq 2$ then leads to the bound

$$\alpha' \geq 0.38 \tag{10}$$

on α'. This indicates a λ transition of the type given by (5) and is definitely different from the commonly accepted value of $\alpha' = 0$ for two-dimensional lattices.[10] This result suggests that the logarithmic singularity of the nearest-neighbor Ising model is indeed a singular case. It must be noted that this is not the first time that the two-dimensional nearest-neighbor model is found to possess a unique behavior. In a recent study on the behavior of two-point correlation functions on a phase boundary, Fisher and Camp[11] showed that the planar nearest-neighbor model is unique in having a decay exponent different from the Ornstein-Zernike form. We feel that these are strong evidences which indicate that the four-spin or the crossing interactions in a planar Ising model will in general lead to a critical exponent $\alpha' \neq 0$.

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[1]R. J. Baxter, Phys. Rev. Letters 26, 832 (1971).

[2]C. Fan and F. Y. Wu, Phys. Rev. B 2, 723 (1970).

[3]The proof follows closely that given in the Appendix of F. Y. Wu, Phys. Rev. 183, 604 (1969), which will not be reproduced here. The readers are also referred to the following review article for a more comprehensive discussion: E. H. Lieb and F. Y. Wu, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, London, 1971).

[4]There exist other mappings between the ferroelectric and the Ising problems. [See, e.g., M. Suzuki and M. E. Fisher, J. Math. Phys. 12, 235 (1971); E. H. Lieb and F. Y. Wu, in Ref. 3.] These mappings would, however, lead to infinite Ising interactions in the present problem.

[5]The zero-energy level has been chosen to make $\epsilon_1 + \epsilon_3 + \epsilon_5 + \epsilon_7 = 0$.

[6]This is the case considered by F. Y. Wu, in Ref. 3.

[7]See E. H. Lieb and F. Y. Wu in Ref. 3.

[8]C. Domb and N. W. Dalton, Proc. Phys. Soc. (London) 89, 859 (1966).

[9]N. W. Dalton and D. W. Wood, J. Math. Phys. 10, 1271 (1969).

[10]We feel that the estimates on γ' in Ref. 9 are sufficient to indicate $\alpha' > 0$.

[11]M. E. Fisher and W. J. Camp, Phys. Rev. Letters 26, 565 (1971).

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ERRATUM

Enhancement of Superconductivity in Aluminum Films, J. J. Hauser [Phys. Rev. B 3, 1611 (1971)]. Figure 1 caption should read: Transition temperature of Al-Ge and Al-Al_2O_3 films as a function of low-temperature resistivity. The values Al-10 wt% (3.6-at.%) Ge and Al-10 wt% Al_2O_3 (7-at.% O) quoted in the caption correspond only to the peak in T_c as explained in the text.